Lab Work:
- Collect a timer tape for a chain sliding off of the edge of the table. Every person collects their own tape. Prior to releasing the chain, measure the length of chain hanging vertically from the end of the table.
- Measure the mass and full length of the chain.
- Drag the chain at constant speed across the lab table.

Next steps:
- Using the outline provided to you, create a mathematical model for the chain sliding off the table. Include handwritten and scanned math in which you perform all mathematical steps and predict the slope of the best fit line of chain acceleration $a$ as a function of position $x$.
- Perform an analysis from the timer tape to characterize the actual motion of the chain, and compare the outcome to your mathematical model.
  - Explain any discrepancies between the experimental outcome and the predicted results by considering both physical differences between the lab setup and what is assumed in the model, and by evaluating sources of error.
- Submit your results in a partial inquiry report including data, analysis, and conclusions. Include math (may be handwritten and scanned) to justify the mathematical model you are using to explain the outcome of the lab.

Chain Lab Derivations

3. $\Sigma F = ma$
   \[ m \left( \frac{L}{L} \right) g = ma \]
   Fraction of chain hanging from table:
   \[ \left( \frac{L}{L} \right) x = ax \]

5. \[
   \frac{g}{L} x \frac{dx}{dt} = g \frac{dv}{dt} \to \frac{g}{L} x \frac{dv}{dx} = \frac{g}{L} v \frac{dv}{dx}
\]
   \[
   \frac{g}{L} \int_0^L x \, dx = \int_0^L v \, dv
\]
   \[
   \frac{g}{L} \left[ \frac{x^2}{2} \right]_0^L = \left[ \frac{v^2}{2} \right]_0^L
\]
   \[
   \frac{g}{L} L^2 = \frac{v^2}{2}
\]
   \[
   v = \sqrt{gL}
\]

6. \[
   \frac{g}{L} \left[ \frac{x^2}{2} \right]_0^L = \frac{v^2}{2}
\]
   \[
   \frac{g}{L} (L^2 - x^2) = \frac{v^2}{2}
\]
   \[
   v = \sqrt{\frac{g}{L} (L^2 - x^2)}
\]

7. $f = \mu_k N$
   $f = \mu_k \left( \frac{L-x}{L} \right) mg$
   $f = \mu_k \left( \frac{L-x}{L} \right) mg$
   Mass of the chain on the table.

8. $\Sigma F = ma$
   \[
   \left( \frac{L}{x} \right) mg - f = ma
   \]
   \[
   \left( \frac{L}{x} \right) mg = \mu_k \left( \frac{L}{L-x} \right) g
   \]
   \[
   a = \left( \frac{L}{x} \right) g - \mu_k g \frac{L}{L-x}
   \]
   \[
   a = \left( \frac{L}{x} \right) x - \mu_k g + \mu_k x
   \]
   \[
   a = \left( \frac{L}{x} \right) (1 + \mu_k) x - \frac{\mu_k g}{x}
   \]
   \[
   y = mx + b
   \]

Our model predicts that acceleration and position will have a linear relationship. This is not consistent with a constant jerk rate, where acceleration and time would have a linear relationship.

Remember:
- Friction is in our model, as is the need to hang part of the chain off of the side of the table.
- Not in our model are the effect of individual links sliding off of the table, and the horizontal velocity the chain acquires as it slides from the table.
Estimates of slope and y-intercept based on data Mr. Wright collected in class.

\[ \mu = \frac{f}{N} = \frac{0.17}{3.7} = 0.02 \]

\[ L = 73.5 \text{ cm} \]

\[ \text{Slope} = \left( \frac{g}{L} \right) (1 + \mu) = \left( \frac{9.8}{73.5} \right) (1 + 0.02) = 0.16 \text{ s}^{-2} \]

\[ y \text{- intercept} = -\mu g = -0.02 (9.8) = -0.196 \text{ cm/s}^2 \]

The y-intercept allows for a prediction of the position at which positive acceleration would occur, that is, of how much chain must be hanging from the table in order for the chain to start sliding. At the boundary, \( a = 0 \), and

\[ x = \frac{\mu g}{(\mu + \mu_K) \left( 1 + \mu_K \right)} = \frac{\mu g}{1 + \mu_K} = \left( \frac{0.02}{1 + 0.02} \right) (73.5) = 12 \text{ cm} \]

The actual value of \( x \) will be larger, since \( \mu_S > \mu_K \).

Expect only rough agreement between the slope calculated above and the slope from your graph.

My data and graph. Note that the maximum position is 34 cm, just at the point where many of you saw large increases in measured acceleration, possibly due to the chain's horizontal motion at the edge of the table.